

Questions of the condition of roots are also briefly considered. The chapter concludes with some interpolatory methods: the regula falsi, secant method, Muller's method, and with their convergence properties, and, finally, with convergence-accelerating processes such as those of Aitken and Steffensen.

Each chapter is followed by a good set of exercises and by a list of selected references. The text makes occasional references to the items in these bibliographies, but additional notes on some of the sources would have been helpful. The index at the end of the book, on the whole, seems adequate, but the term "good-natured algorithm" is conspicuously missing.

Unfortunately, the book is not error-free; in fact, there are quite a few of them. However, most of the errors are of a trivial nature and can easily be rectified by an alert reader.

From the above outline of content, it should be apparent that we have here before us a text which is thoroughly up-to-date, original both in the selection of topics and in their mathematical treatment, a book, in short, which contains the essence of a good many years of experience in computing. Adding to this the extreme clarity and conciseness of exposition makes this indeed one of the outstanding introductory texts in numerical analysis. It is only to be hoped that an English translation will be available in the not too distant future.

W. G.

1. A. H. STROUD & D. SECREST, *Gaussian Quadrature Formulas*, Prentice-Hall, Englewood Cliffs, N. J., 1964.

18 [2.10].—JAMES L. PHILLIPS & RICHARD J. HANSON, *Gauss Quadrature Rules with B-Spline Weight Functions*, 28 pages of tables and 4 pages of explanatory text, reproduced on the microfiche card attached to this issue.

The abscissas and weights of n -point Gaussian quadrature rules for integrals

$$\int_{-1}^1 N(i, k; t)y(t) dt$$

are tabulated to 14S for $n = 1(1)17$, $k = 2, 4$, $i = 1(1)k$. Here $N(i, k; t)$ is a normalized B -spline of order k (degree $k - 1$) with support on $(-1, 1)$. Translates and reflections of the k B -splines $N(1, k; t), \dots, N(k, k; t)$ provide a basis for the space of splines of order k defined on an interval $[a, b]$ with respect to a partition of equally spaced interior knots and end knots of multiplicity k .

The first 17 coefficients in the three term recurrence formula for polynomials orthonormal on $(-1, 1)$ with respect to the weight function $N(i, k; t)$ are given to 14S for the same values of i and k .

Details of the underlying calculations on an IBM 360/67 at Washington State University are also furnished.

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Gauss Quadrature Rules with B-Spline Weight Functions

by James L. Phillips and Richard J. Hanson

B-splines are nonnegative polynomial spline basis functions with finite support which are useful for representing splines in numerical work (deBoor [1]). The tables presented here contain Gaussian quadrature rules with certain normalized B-splines of order k (degree $k-1$) as weight functions, where $k = 2, 4$. These functions are translates and reflections of B-splines defined over any finite interval $[a, b]$ with respect to a partition of equally spaced interior knots and end knots of multiplicity k .

Given $[a, b]$, define a partition τ ,

$$\tau: t_1 = \dots = t_k = a < t_{k+1} < \dots < t_n < b = t_{n+1} = \dots = t_{n+k}$$

where $t_{j+1} - t_j = h = \text{constant}$ for $j = k, \dots, n$. The normalized B-splines $\check{N}(1, k; t), \dots, \check{N}(n, k; t)$ are a basis for the space of splines of order k with knots on τ . Here (deBoor [1])

$$\check{N}(i, k; t) = (t_{i+k} - t_i) G_k(t_i, \dots, t_{i+k}; t), \quad i = 1, \dots, n, \text{ where}$$

$$G_k(t_i, \dots, t_{i+k}; t) \text{ is the } k\text{-th divided difference (in } x \text{ for fixed } t)$$

$$\text{of } G_k(x; t) = (x - t)_+^{k-1} = \begin{cases} (x-t)^{k-1}, & x > t \\ 0 & , x \leq t. \end{cases}$$

Because of the particular nature of the partition τ , each of the last $n-k$ B-splines $\check{N}(i, k; t)$ is a translate or reflection of one of the first k of these functions. In particular,

$$(1) \quad \check{N}(j, k; t) = \check{N}(n+1-j, k; b+a-t), \quad j > n+1-k$$

and

$$(2) \quad \check{N}(j, k; t) = \check{N}(k, k; t - (j-k)h), \quad k \leq j \leq n+1-k.$$

We define translates $N(i, k; t)$ of the first k B-splines $\check{N}(i, k; t)$ such that each $N(i, k; t)$ has its support on $(-1, 1)$. Thus,

$$(3) \quad N(i, k; t) = \hat{N}(i, k; \alpha_i t + \beta_i), \quad i=1, \dots, k,$$

where

$$(4) \quad \alpha_i = h(t_{i+k} - t_i), \quad \beta_i = h(t_{i+k} + t_i), \quad i=1, \dots, k.$$

If $s(t) = \sum_{i=1}^n c_i N(i, k; t)$ and $f(t)$ is a given function, then equations (1), (2), and (3) imply

$$\int_a^b s(t) f(t) dt = \sum_{i=1}^k \alpha_i^{-1} \int_{-1}^1 N(i, k; t) f_i(t) dt,$$

where

$$f_i(t) = \begin{cases} c_i f(\alpha_i t + \beta_i) + c_{n+1-i} f(b + a - \alpha_i t - \beta_i), & i < k \\ \sum_{j=k}^{n-k+1} c_j f(\alpha_k t + \beta_k + (j-k)h), & i = k. \end{cases}$$

Alternatively, if we define α_i and β_i as in (4) for all $i=1, \dots, n$, we can write

$$\int_a^b s(t) f(t) dt = \sum_{i=1}^n c_i \alpha_i^{-1} \int_{-1}^1 N(m_i, k; t) f(\alpha_i t + \beta_i) dt,$$

where

$$m_i = \begin{cases} 1, & i < k \\ k, & k \leq i \leq n-k+1 \\ n+1-i, & i > n-k+1 \end{cases}, \quad \sigma_i = \begin{cases} \alpha_i, & i \leq n-k+1 \\ -\alpha_i, & i > n-k+1. \end{cases}$$

The tables which follow contain Gaussian quadrature rules for evaluating integrals of the form

$$\int_{-1}^1 N(i, k; t) y(t) dt, \quad 1 \leq i \leq k, \quad k = 2, 4,$$

using $N(i, k; t)$ as a weight function. Preceding the rules for each order are the first 17 coefficients a_j, b_j in the recurrence formulas

$$(5) \quad \begin{cases} p_0(t) = b_0 \\ b_1 p_1(t) = t - a_0 \\ b_j p_j(t) = (t - a_{j-1}) p_{j-1}(t) - b_{j-1} p_{j-2}(t), & j > 1, \end{cases}$$

for the polynomials $p_j(t) = p(i, k, j; t)$ orthonormal on $(-1, 1)$ with respect to the weight function $N(i, k; t)$.

The tables were computed using double precision arithmetic on an IBM 360/67 computer at Washington State University. The coefficients a_j and b_j of (5) were found first, using the fact that a_j and b_j^2 are coefficients of related monic orthogonal polynomials

$$\begin{cases} q_0(t) = 1 \\ q_1(t) = t - a_0 \\ q_j(t) = (t - a_{j-1})q_{j-1}(t) - b_{j-1}^2 q_{j-2}(t), \quad j > 1, \end{cases}$$

where

$$\begin{aligned} a_j &= (tq_j, q_j) / (q_j, q_j), \quad j > 0 \\ b_j &= (q_j, q_j) / (q_{j-1}, q_{j-1}), \quad j > 1. \end{aligned}$$

The inner products (tq_j, q_j) and (q_j, q_j) were evaluated to near machine accuracy by dividing each integrand into a sum of integrals of polynomials over single knot subintervals. A Gauss-Legendre quadrature rule of sufficient order to assure exact quadrature was then applied to evaluate each summand.

The quadrature abscissas e_j were computed using the procedure of Golub and Welsh [2]. The weights w_i were computed from the formula

$$w_i^{-1} = \sum_{j=0}^{m-1} p_j(e_i)^2.$$

Here m denotes the number of points in the quadrature rule being computed.

Further details regarding the computations are given in Phillips and Hanson [3].

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References

1. C. R. deBoor, "On calculating with B-splines," J. Approximation Theory, v. 6, 1972, pp. 50-62.
2. G. H. Golub & J. H. Welsh, "Calculation of Gauss quadrature rules," Math. Comp., v. 23, 1969, pp. 221-230.
3. J. L. Phillips & R. J. Hanson, Computing Integrals Involving B-Splines by Means of Specialized Gaussian Quadrature Rules, Technical Report #CS-73-001, Computer Science Dept., Washington State University, Pullman, Washington, 1973.

COEFFICIENTS OF THE THREE TERM RECURRENCE RELATION
 FOR POLYNOMIALS $P(I, K, J; T)$ ORTHONORMAL WITH RESPECT
 TO THE B-SPLINE $N(I, K; T)$, $K=2$

J	A(J)	B(J)
I = 1		
0	-0.333333333333333D 00	0.100000000000000D 01
1	-0.666666666666667D-01	0.47140452079103D 00
2	-0.285714285714290-01	0.48989794855664D 00
3	-0.158730158730160-01	0.49487165930539D 00
4	-0.101010101010100D-01	0.49690399499995D 00
5	-0.699300699300660D-02	0.49792959773197D 00
6	-0.512820512820570D-02	0.49851851526214D 00
7	-0.392156862745040D-02	0.49888765156985D 00
8	-0.30959752321985D-02	0.49913419848462D 00
9	-0.25062656641599D-02	0.49930699897395D 00
10	-0.207039337474190D-02	0.49943278484293D 00
11	-0.17391304347821D-02	0.49952718665548D 00
12	-0.14814814814818D-02	0.49959983987187D 00
13	-0.12771392081741D-02	0.49965694678637D 00
14	-0.11123470522796D-02	0.49970264642033D 00
15	-0.97751710655022D-03	0.49973978660741D 00
16	-0.86580086579992D-03	0.49977037886275D 00
I = 2		
0	0.0	0.100000000000000D 01
1	0.0	0.40824829046386D 00
2	0.0	0.48304589153965D 00
3	0.0	0.48234122903240D 00
4	0.0	0.49450977870175D 00
5	0.0	0.49247621885964D 00
6	0.0	0.49733337563629D 00
7	0.0	0.49582714426470D 00
8	0.0	0.49843460197895D 00
9	0.0	0.49734543938561D 00
10	0.0	0.49897360685953D 00
11	0.0	0.49816188392322D 00
12	0.0	0.49927634846718D 00
13	0.0	0.49865150629245D 00
14	0.0	0.49946292095026D 00
15	0.0	0.49896829781477D 00
16	0.0	0.49958586144845D 00

ABSCISSAS

WEIGHTS

*** 1 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=2 ***

	I = 1	
-0.333333333333333D 00		0.100000000000000D 01
	I = 2	
0.0		0.100000000000000D 01

*** 2 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=2 ***

	I = 1	
-0.68989794855664D 00		0.63609276348795D 00
0.28989794855664D 00		0.36391723651205D 00
	I = 2	
-0.40824829046386D 00		0.50000000000000D 00
0.40824829046386D 00		0.50000000000000D 00

*** 3 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=2 ***

	I = 1	
-0.82282408097459D 00		0.40186382747792D 00
-0.18106627111853D 00		0.45848221271917D 00
0.57531892352169D 00		0.13965395980291D 00
	I = 2	
-0.63245553203368D 00		0.20833333333333D 00
0.0		0.58333333333333D 00
0.63245553203368D 00		0.20833333333333D 00

*** 4 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=2 ***

	I = 1	
-0.88579160777096D 00		0.27101382686298D 00
-0.44631397272375D 00		0.40692913602054D 00
0.16718086473783D 00		0.25969509521646D 00
0.72049027131244D 00		0.62361941900017D-01
	I = 2	
-0.75092514330445D 00		0.98866304579876D-01
-0.26222984265275D 00		0.40113369542012D 00
0.26222984265275D 00		0.40113369542012D 00
0.75092514330445D 00		0.98866304579876D-01

ABSCISSAS

WEIGHTS

*** 5 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=2 ***

I = 1
 -0.920380285897060 00 0.193563180453300 00
 -0.603973164252780 00 0.334349276188740 00
 -0.124050379505230 00 0.292773974169340 00
 0.390928546707270 00 0.147817740145230 00
 0.802929828402350 00 0.314958290433850-01

I = 2
 -0.821440599738380 00 0.516582577654910-01
 -0.449920352459840 00 0.239473240705460 00
 0.0 0.417737003058100 00
 0.449920352459840 00 0.239473240705460 00
 0.821440599738380 00 0.516582577654910-01

*** 6 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=2 ***

I = 1
 -0.941367145680430 00 0.144620661451020 00
 -0.703842800663030 00 0.271084994463040 00
 -0.326030619437690 00 0.281585107576400 00
 0.117343037543100 00 0.197322301781310 00
 0.538467724060110 00 0.879103311010180-01
 0.853891342639480 00 0.174766036272200-01

I = 2
 -0.865738294978820 00 0.294965616553090-01
 -0.576614808248490 00 0.147481233705270 00
 -0.194263676775040 00 0.323022204639420 00
 0.194263676775040 00 0.323022204639420 00
 0.576614808248490 00 0.147481233705270 00
 0.865738294978820 00 0.294965616553100-01

*** 7 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=2 ***

I = 1
 -0.955041227122570 00 0.111934726846980 00
 -0.770641893678190 00 0.221018516381750 00
 -0.468420354430820 00 0.254781794599180 00
 -0.943072526611110-01 0.214250131391750 00
 0.294750565773660 00 0.132769392930980 00
 0.639518616526210 00 0.549167134437470-01
 0.887474878926150 00 0.104287244056150-01

I = 2
 -0.895615660710820 00 0.179407450256320-01
 -0.665993283705960 00 0.939250393193440-01
 -0.346413302656910 00 0.224746555018040 00
 0.0 0.326775321273980 00
 0.346413302656910 00 0.224746555018040 00
 0.665993283705950 00 0.939250393193450-01
 0.895615660710820 00 0.179407450256320-01

ABSCISSAS

WEIGHTS

*** 8 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=2 ***

I = 1

-0.964440169705270 00	0.891016087231120-01
-0.817352784200410 00	0.182238047272750 00
-0.571383041208740 00	0.225011598941770 00
-0.256135670833460 00	0.212094718871860 00
0.903733696068520-01	0.158399198984640 00
0.426350485711140 00	0.908786390093980-01
0.711267485915710 00	0.356859053119730-01
0.910732089420060 00	0.659038288449800-02

I = 2

-0.916498601781820 00	0.115294486060360-01
-0.730126110448890 00	0.622725860695430-01
-0.464700844889640 00	0.157584233713380 00
-0.154629080840260 00	0.268613731611040 00
0.154629080840260 00	0.268613731611040 00
0.464700844889640 00	0.157584233713380 00
0.730126110448880 00	0.622725860695440-01
0.916498601781820 00	0.115294486060360-01

*** 9 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=2 ***

I = 1

-0.971175180702250 00	0.725560070465600-01
-0.851225220581610 00	0.152148510218620 00
-0.647766687674010 00	0.197067484344690 00
-0.380664840144720 00	0.200617618386740 00
-0.760591978379780-01	0.168716643689840 00
0.236234469390590 00	0.116802390590330 00
0.525646030370080 00	0.636096429821080-01
0.763842042420000 00	0.241200085695710-01
0.927484374233580 00	0.436169417154660-02

I = 2

-0.931753814324370 00	0.772467225704890-02
-0.777874522766330 00	0.426373772293870-01
-0.554352720106100 00	0.111965186394200 00
-0.284230579976570 00	0.203250033165550 00
0.0	0.268845461907630 00
0.284230579976570 00	0.203250033165550 00
0.554352720106100 00	0.111965186394200 00
0.777874522766330 00	0.426373772293870-01
0.931753814324370 00	0.772467225704950-02

ABSCISSAS

WEIGHTS

*** 10 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=2 ***

I = 1

-0.976164773135170 00	0.601990160480750-01
-0.876535856245700 00	0.128574309018160 00
-0.70577100713860 00	0.172422600578350 00
-0.477680647983090 00	0.185393787355450 00
-0.210720306228430 00	0.169114219381650 00
0.734775314313200-01	0.132106151126700 00
0.351888923353330 00	0.868038128143020-01
0.601957842073800 00	0.455491829065250-01
0.803421975580290 00	0.168386395659670-01
0.939941935677020 00	0.299828120481310-02

I = 2

-0.943166381725670 00	0.536905972447170-02
-0.814053288655820 00	0.301132562934670-01
-0.623784313633470 00	0.812306244549690-01
-0.388351321334550 00	0.153837064198190 00
-0.128573681331640 00	0.229449995328900 00
0.128573681331640 00	0.229449995328900 00
0.388351321334550 00	0.153837064198190 00
0.623784313633460 00	0.812306244549700-01
0.814053288655820 00	0.301132562934670-01
0.943166381725660 00	0.536905972447210-02

*** 11 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=2 ***

I = 1

-0.979963439076640 00	0.507346813762830-01
-0.895929097745640 00	0.109876182265740 00
-0.750761549711110 00	0.151240096114370 00
-0.554318785912320 00	0.169318845768030 00
-0.319983684170570 00	0.163758205976130 00
-0.637247738208320-01	0.139063750316370 00
0.196994559534280 00	0.103182721344600 00
0.444406569781930 00	0.652830934276670-01
0.661649799245630 00	0.333272469033620-01
0.833916773105190 00	0.120878419209600-01
0.949452759204960 00	0.212733458648940-02

I = 2

-0.951962495353780 00	0.384214878746310-02
-0.842214930806630 00	0.218090146209440-01
-0.678751409215900 00	0.600039913499350-01
-0.473064566856190 00	0.117037877707950 00
-0.240047721802410 00	0.183013810568850 00
0.0	0.228586313929710 00
0.240047721802410 00	0.183013810568850 00
0.473064566856190 00	0.117037877707950 00
0.678751409215900 00	0.600039913499360-01
0.842214930806620 00	0.218090146209450-01
0.951962495393770 00	0.384214878746350-02

ABSCISSAS

WEIGHTS

*** 12 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=2 ***

I = 1

-0.982921890023140 00	0.433297217737450-01
-0.911107073689180 00	0.948557039607550-01
-0.786291018233050 00	0.133213501253350 00
-0.615697890940290 00	0.153793205360080 00
-0.409238231474840 00	0.155392633631060 00
-0.178909837597090 00	0.140218679995260 00
0.619016986256330-01	0.113227687427330 00
0.299201300554510 00	0.809033074138270-01
0.519197779050450 00	0.497535608185310-01
0.709105087529870 00	0.248720183328450-01
0.857884202528820 00	0.888961559133740-02
0.956875873668290 00	0.155036444187640-02

I = 2

-0.958856749716330 00	0.282212292053920-02
-0.864446305250020 00	0.161689888114440-01
-0.722662954737100 00	0.451701842099210-01
-0.541959953882950 00	0.900991324673090-01
-0.333288352107200 00	0.145626609798660 00
-0.110105263381640 00	0.200112961792130 00
0.110105263381640 00	0.200112961792130 00
0.333288352107200 00	0.145626609798660 00
0.541959953882950 00	0.900991324673100-01
0.722662954737100 00	0.451701842099220-01
0.864446305250010 00	0.161689888114450-01
0.958856749716330 00	0.282212292053970-02

*** 13 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=2 ***

I = 1

-0.985270697947820 00	0.374294631712310-01
-0.923203722520640 00	0.826405788396790-01
-0.814809550601990 00	0.117907875913290 00
-0.665497977216880 00	0.139426936564290 00
-0.482752918588470 00	0.145699393031640 00
-0.275737205435520 00	0.137631058975130 00
-0.548312279917660-01	0.118240962628180 00
0.168687928042680 00	0.919915613521050-01
0.384202003439200 00	0.638735743244070-01
0.580314056546870 00	0.384279143294400-01
0.747389642613370 00	0.188978489591860-01
0.877048918201460 00	0.667671950154800-02
0.962779269978020 00	0.115611340987270-02

ABSCISSAS

WEIGHTS

13 POINT RULES, K=2 (CONTINUED)

I = 2

-0.964377258915660 00	0.211776970537790-02
-0.882351791792810 00	0.122225750125350-01
-0.758372949899710 00	0.345533381045740-01
-0.598802694757980 00	0.701108106143460-01
-0.411921702437300 00	0.116050389590580 00
-0.207786708889280 00	0.165480235392810 00
0.0	0.198929763159550 00
0.207786708889280 00	0.165480235392810 00
0.411921702437300 00	0.116050389590580 00
0.598802694757980 00	0.701108106143460-01
0.758372949899710 00	0.345533381045750-01
0.882351791792810 00	0.122225750125360-01
0.964377258915650 00	0.211776970537830-02

*** 14 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=2 ***

I = 1

-0.987166478414360 00	0.326535086451940-01
-0.932597190935970 00	0.725921561108870-01
-0.838029000636090 00	0.104891907003830 00
-0.706390264637570 00	0.126426051148660 00
-0.543831458701480 00	0.135653319208370 00
-0.357456512022130 00	0.132784701252170 00
-0.155410685384860 00	0.119571118045060 00
0.534757226797440-01	0.990389090464060-01
0.260073376740810 00	0.748861929179720-01
0.455352905778530 00	0.508469308142980-01
0.630779478886950 00	0.300646074978580-01
0.778685617639030 00	0.145929682664260-01
0.892605400120550 00	0.510820263537570-02
0.967550468197200 00	0.879427417494560-03

I = 2

-0.968852933897570 00	0.162038441456210-02
-0.896932969804890 00	0.940730040794060-02
-0.787664486184170 00	0.268495328737350-01
-0.645920907268500 00	0.552272581915490-01
-0.478068553810550 00	0.931370635537190-01
-0.291805201436570 00	0.136381124140400 00
-0.963159988009890-01	0.177377336418100 00
0.963159988009900-01	0.177377336418100 00
0.291805201436570 00	0.136381124140400 00
0.478068553810550 00	0.931370635537200-01
0.645920907268500 00	0.552272581915500-01
0.787664486184170 00	0.268495328737360-01
0.896932969804890 00	0.940730040794130-02
0.968852933897570 00	0.162038441456260-02

ABSCISSAS

WFIGHTS

*** 15 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=2 ***

I = 1

-0.988718622054980	00	0.287341392434260	-01
-0.941035402704110	00	0.642380897499320	-01
-0.857174093769680	00	0.937832798881190	-01
-0.740337948892820	00	0.114797651013880	00
-0.595014489899790	00	0.125836213297190	00
-0.426787827484950	00	0.126687910323020	00
-0.242122622706040	00	0.118348384119430	00
-0.481153830735320	-01	0.102824724623770	00
0.147778321813370	00	0.828014703688450	-01
0.338030390059920	00	0.612195391729370	-01
0.515329478062680	00	0.408326185452400	-01
0.672861921211210	00	0.238083807101960	-01
0.804573401358750	00	0.114344447894430	-01
0.905400819811660	00	0.397246938629050	-02
0.971461090526340	00	0.680584768276080	-03

I = 2

-0.972540410786680	00	0.126027296631780	-02
-0.908991047151670	00	0.735180913306670	-02
-0.812034859198470	00	0.211454544478920	-01
-0.685469162792660	00	0.439720941716040	-01
-0.534275272730070	00	0.752507401463210	-01
-0.364492504572650	00	0.112367711425360	00
-0.183195815443060	00	0.150576723159420	00
0.0		0.176150389100030	00
0.183195815443050	00	0.150576723159420	00
0.364492504572640	00	0.112367711425360	00
0.534275272730070	00	0.752507401463210	-01
0.685469162792660	00	0.439720941716050	-01
0.812034859198460	00	0.211454544478930	-01
0.908991047151670	00	0.735180913306670	-02
0.972540410786670	00	0.126027296631810	-02

ABSCISSAS

WEIGHTS

*** 16 POINT RULES, WEIGHT FUNCTION $N(I,K;T)$, $K=2$ ***

I = 1

-0.990005400672460 00	0.254786213146700-01
-0.947712972637210 00	0.572249051802860-01
-0.673138108832330 00	0.842588623148560-01
-0.768803124840370 00	0.104457188807920 00
-0.638258880684230 00	0.116508480419710 00
-0.485950384309660 00	0.120001709776230 00
-0.317064144904350 00	0.115437759218530 00
-0.137351312808750 00	0.104128275687610 00
0.470682320503330-01	0.879947643784870-01
0.229914303752970 00	0.692956331585720-01
0.404960275850080 00	0.503184662129400-01
0.566245107929730 00	0.330791687112720-01
0.708276295550420 00	0.190683357693320-01
0.826216753074280 00	0.907846477227780-02
0.916048835380940 00	0.313437694719210-02
0.974706041252080 00	0.534997330111350-03

I = 2

-0.975607556501850 00	0.995016039794430-03
-0.919050168276020 00	0.582755324380930-02
-0.832462009330440 00	0.168686405891930-01
-0.718843447524200 00	0.353953008500040-01
-0.582143181778260 00	0.613026994963560-01
-0.427159902199410 00	0.929896769015520-01
-0.259468330413810 00	0.127358890722370 00
-0.856200111207560-01	0.159262222156920 00
0.856200111207560-01	0.159262222156920 00
0.259468330413810 00	0.127358890722370 00
0.427159902199410 00	0.929896769015530-01
0.582143181778250 00	0.613026994963570-01
0.718843447524190 00	0.353953008500050-01
0.832462009330430 00	0.168686405891940-01
0.919050168276010 00	0.582755324380990-02
0.975607556501850 00	0.995016039794870-03

ABSCISSAS

WEIGHTS

*** 17 POINT RULES, WEIGHT FUNCTION $N(I,K;T)$, $K=2$ ***

I = 1

-0.991384012864430 00	0.227454062263940-01
-0.953319811752450 00	0.512848558404100-01
-0.886584062461840 00	0.760514576514850-01
-0.792889134129610 00	0.952837387492360-01
-0.675079993143730 00	0.107815863335050 00
-0.536735748445660 00	0.113147222764940 00
-0.382059764495610 00	0.111469835847120 00
-0.215751730555810 00	0.103619483858590 00
-0.428648062938430-01	0.909555807982840-01
0.131347933773270 00	0.751846509742240-01
0.301593116906760 00	0.581490481754100-01
0.462697905468100 00	0.416065560205910-01
0.609767157049930 00	0.270273949046020-01
0.738332119134820 00	0.154328444296120-01
0.844486069184590 00	0.729446096684330-02
0.925002395646690 00	0.250536687878280-02
0.977428081264480 00	0.426232578414120-03

I = 2

-0.978190923548550 00	0.795786785436240-03
-0.927544053020900 00	0.467618953961930-02
-0.849781015434220 00	0.136078219471840-01
-0.747304226317450 00	0.287661571111070-01
-0.623285334002250 00	0.503120299057060-01
-0.481592280569340 00	0.772851447682220-01
-0.326721215483840 00	0.107595096007750 00
-0.163828556798710 00	0.137915746626650 00
0.0	0.158092054616660 00
0.163828556798710 00	0.137915746626650 00
0.326721215483840 00	0.107595096007750 00
0.481592280569340 00	0.772851447682230-01
0.623285334002250 00	0.503120299057070-01
0.747304226317440 00	0.287661571111080-01
0.849781015434210 00	0.136078219471850-01
0.927544053020900 00	0.467618953961980-02
0.978190923548540 00	0.795786785436550-03

COEFFICIENTS OF THE THREE TERM RECURRENCE RELATION
 FOR POLYNOMIALS P(I,K;J;T) ORTHONORMAL WITH RESPECT
 TO THE B-SPLINE N(I,K;T), K=4

J	A(J)	B(J)
I = 1		
0	-0.6000000000000000 00	0.141421356237310 01
1	-0.257142857142860 00	0.326598632371090 00
2	-0.142857142857140 00	0.412393049421160 00
3	-0.909090909090910-01	0.447213595499960 00
4	-0.629370629370630-01	0.464734291216500 00
5	-0.461538461538460-01	0.474779538344900 00
6	-0.352941176470590-01	0.481070235442360 00
7	-0.278637770897830-01	0.485269359637830 00
8	-0.225563909774440-01	0.488211287885640 00
9	-0.186335403726710-01	0.490352188754880 00
10	-0.156521739130430-01	0.491958592918280 00
11	-0.133333333333340-01	0.493194713719670 00
12	-0.114942528735640-01	0.494166211107400 00
13	-0.100111234705220-01	0.49494357359720 00
14	-0.879765395894480-02	0.495575288385680 00
15	-0.779220779220770-02	0.496095596665230 00
16	-0.694980694980680-02	0.496529236984030 00
I = 2		
0	-0.4000000000000000 00	0.141421356237310 01
1	-0.207142857142860 00	0.326598632371090 00
2	-0.114732142857140 00	0.412393049421160 00
3	-0.763478903534090-01	0.445452280693450 00
4	-0.525368194150400-01	0.463461931324450 00
5	-0.393845383326790-01	0.473413803841400 00
6	-0.299818934733320-01	0.480028913696100 00
7	-0.239838725046700-01	0.484286425622450 00
8	-0.193535645171720-01	0.487423550309430 00
9	-0.161278671885020-01	0.489627690309730 00
10	-0.135162356439660-01	0.491356193504340 00
11	-0.115851781846410-01	0.492643031171740 00
12	-0.997014551147050-02	0.493694847160030 00
13	-0.872361654198620-02	0.494510994966680 00
14	-0.765617193157970-02	0.495197990285140 00
15	-0.680520902439020-02	0.495747930704440 00
16	-0.606326165602310-02	0.496221101345530 00

J	A(J)	B(J)
		I = 3
0	-0.200000000000000 00	0.141421356237310 01
1	-0.119327731092440 00	0.317396819046350 00
2	-0.699877402357770-01	0.400112708949360 00
3	-0.447395169649280-01	0.438680332728830 00
4	-0.328246860577280-01	0.456722121890410 00
5	-0.237159167379160-01	0.468686846604530 00
6	-0.185424181491130-01	0.476002175601660 00
7	-0.147924859501020-01	0.480948232312710 00
8	-0.118931127057090-01	0.484692909700750 00
9	-0.100136323411520-01	0.487253442811980 00
10	-0.835642525746850-02	0.489326167663420 00
11	-0.716364014805650-02	0.490902108080080 00
12	-0.621287090642400-02	0.492126948058280 00
13	-0.538451034662740-02	0.493167175651270 00
14	-0.477573423058340-02	0.493970998334290 00
15	-0.421819750893720-02	0.494664658006430 00
16	-0.376937514450160-02	0.495242265756700 00

J	A(J)	B(J)
		I = 4
0	0.0	0.141421356237310 01
1	0.0	0.288675134594810 00
2	0.0	0.376386326354540 00
3	0.0	0.422328479430190 00
4	0.0	0.445889772948450 00
5	0.0	0.458863457070840 00
6	0.0	0.468930108681890 00
7	0.0	0.475142361270480 00
8	0.0	0.479614007212730 00
9	0.0	0.483175915057040 00
10	0.0	0.485821423090070 00
11	0.0	0.487763261628990 00
12	0.0	0.489503972986470 00
13	0.0	0.490799748599930 00
14	0.0	0.491870534244660 00
15	0.0	0.492802773462110 00
16	0.0	0.493568776614830 00

ABSCISSAS

WEIGHTS

*** 1 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=4 ***

	I = 1	
-0.6000000000000000 00		0.5000000000000000 00
	I = 2	
-0.4000000000000000 00		0.5000000000000000 00
	I = 3	
-0.2000000000000000 00		0.5000000000000000 00
	I = 4	
0.0		0.5000000000000000 00

*** 2 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=4 ***

	I = 1	
-0.797426985353090 00		0.366189500386220 00
-0.597158717897700-01		0.133810499613780 00
	I = 2	
-0.644107970009230 00		0.320791647660950 00
0.369651128663760-01		0.179208352339050 00
	I = 3	
-0.479613462332050 00		0.281517569375760 00
0.160285731239610 00		0.218482430624240 00
	I = 4	
-0.288675134594810 00		0.2500000000000000 00
0.288675134594810 00		0.2500000000000000 00

*** 3 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=4 ***

	I = 1	
-0.875848201239750 00		0.257842086321040 00
-0.397622538327230 00		0.209199795113610 00
0.273470739566980 00		0.329581185653440-01
	I = 2	
-0.763150261407780 00		0.182718033752690 00
-0.284557740300590 00		0.268580712626030 00
0.325835001708370 00		0.487012536212840-01
	I = 3	
-0.634633813240450 00		0.131001567429720 00
-0.149323007734550 00		0.298703422317750 00
0.394641349646790 00		0.702950102525260-01
	I = 4	
-0.474341649025260 00		0.925925925925930-01
0.0		0.314814814814810 00
0.474341649025260 00		0.925925925925930-01

ABSCISSAS

WEIGHTS

*** 4 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=4 ***

I = 1

-0.915694161132240 00	0.187591079717050 00
-0.580566459938570 00	0.218008737877280 00
-0.716928921765010-01	0.850934482855330-01
0.477044422338220 00	0.931673412013910-02

I = 2

-0.830548124333470 00	0.106770194118260 00
-0.472939407368800 00	0.250052880411010 00
-0.339251792673900-02	0.128679551194460 00
0.508657159275600 00	0.144973742762680-01

I = 3

-0.730547466868970 00	0.612723306522280-01
-0.348318402371210 00	0.241288101896890 00
0.935322457242240-01	0.174849621992490 00
0.551278635222820 00	0.225899454583920-01

I = 4

-0.601947432461460 00	0.329209899198790-01
-0.202535510691020 00	0.217079010080120 00
0.202535510691020 00	0.217079010080120 00
0.601947432461460 00	0.329209899198790-01

*** 5 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=4 ***

I = 1

-0.938871406986530 00	0.141521056193510 00
-0.691210299947680 00	0.200633008929920 00
-0.297140008474120 00	0.120899906407450 00
0.165799576355600 00	0.339146497265440-01
0.607575985206580 00	0.304137874258020-02

I = 2

-0.872670610710530 00	0.652607313753870-01
-0.596380319954190 00	0.198817250752700 00
-0.220327985016810 00	0.176844336432900 00
0.210538750544890 00	0.541741401398640-01
0.628080455368200 00	0.490354129915220-02

I = 3

-0.793294133912010 00	0.303019257056900-01
-0.486205891858130 00	0.162674931581870 00
-0.118386615277530 00	0.214371958248620 00
0.274669458849650 00	0.846284406287620-01
0.656337507847140 00	0.802284383505910-02

I = 4

-0.690884085017230 00	0.124008275408520-01
-0.353352819753490 00	0.119448769541670 00
0.0	0.236300805834950 00
0.353352819753490 00	0.119448769541670 00
0.690884085017230 00	0.124008275408520-01

ABSCISSAS

WEIGHTS

*** 6 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=4 ***

I = 1

-0.953590703364440 00	0.110165015920890 00
-0.763233368874520 00	0.176601982481670 00
-0.451205556659170 00	0.136886563535280 00
-0.648231334580830-01	0.610523844516330-01
0.337399085924880 00	0.141706318634580-01
0.695453670431320 00	0.112342174707230-02

I = 2

-0.900763341733840 00	0.17347763425030-01
-0.681401940456240 00	0.149919793154390 00
-0.373394528361030 00	0.184627361805070 00
-0.125042071758150-01	0.985158617060080-01
0.368440999131130 00	0.233422965104480-01
0.709476770494670 00	0.185971048158480-02

I = 3

-0.836546310412370 00	0.158932748050770-01
-0.586249326955950 00	0.102946417079460 00
-0.275698985387150 00	0.196417552776430 00
0.660358064446310-01	0.142627736037840 00
0.412735420972440 00	0.389577045292870-01
0.729127804249610 00	0.315731477190270-02

I = 4

-0.753641202985530 00	0.510797376077890-02
-0.468697180904020 00	0.605855086966060-01
-0.158374859646320 00	0.184306517542610 00
0.158374859646320 00	0.184306517542610 00
0.468697180904020 00	0.605855086966070-01
0.753641202985530 00	0.510797376077890-02

*** 7 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=4 ***

I = 1

-0.963539829027850 00	0.880170085309020-01
-0.812725068290370 00	0.153113122726900 00
-0.560507103932170 00	0.139500196214440 00
-0.237208056863180 00	0.819903372075880-01
0.118267078754320 00	0.306243342524880-01
0.463273119377020 00	0.629519281383380-02
0.757145742335150 00	0.459809253947600-03

I = 2

-0.920464729695130 00	0.277759584737120-01
-0.742380553800500 00	0.111454744210320 00
-0.486551667005190 00	0.168971302653630 00
-0.179597136910950 00	0.129235783083310 00
0.156040170008120 00	0.511816389880340-01
0.485670783860560 00	0.106037416731560-01
0.767156991968620 00	0.776830917945080-03

ABSCISSAS

WEIGHTS

7 POINT RULES, K=4 (CONTINUED)

I = 3

-0.867566419013880 00	0.880283874341940-02
-0.660547803936740 00	0.644791184219830-01
-0.395628193561440 00	0.156258722647620 00
-0.978932312342090-01	0.167063266385610 00
0.213204525175200 00	0.837005963777330-01
0.517909326361160 00	0.183385915609580-01
0.781383786972010 00	0.135686586267060-02

I = 4

-0.799357961717730 00	0.227811671600920-02
-0.558838756561470 00	0.302378646828420-01
-0.285230163945820 00	0.122109491997840 00
0.0	0.190747053206620 00
0.285230163945820 00	0.122109491997840 00
0.558838756561470 00	0.302378646828420-01
0.799357961717730 00	0.227811671600920-02

*** 5 POINT RULES, WEIGHT FUNCTION W(I,K;T), K=4 ***

I = 1

-0.970586880170810 00	0.718538936064620-01
-0.848187425797800 00	0.132364070643200 00
-0.640567899357110 00	0.134723678785060 00
-0.367986496863240 00	0.949087596232200-01
-0.570908624038100-01	0.472631613367800-01
0.261752199975670 00	0.157101476825900-01
0.557512985300410 00	0.297136816664640-02
0.801996484579850 00	0.204920156039660-03

I = 2

-0.934815203522800 00	0.191385558190620-01
-0.787504824843060 00	0.830455478409010-01
-0.572297773535920 00	0.145063436614060 00
-0.308324228891850 00	0.141089103387170 00
-0.147885548913320-01	0.794106495929770-01
0.290057668646420 00	0.268079745779330-01
0.574178144329310 00	0.509275917064410-02
0.809384758630090 00	0.351922997251270-03

I = 3

-0.890541463295700 00	0.511704287274740-02
-0.716909055522150 00	0.408282969004720-01
-0.489432207188090 00	0.115813543099770 00
-0.228560688354090 00	0.162008696008920 00
0.509582522250720-01	0.119897954821890 00
0.332202219748490 00	0.466589400339160-01
0.598351421765040 00	0.904636525740160-02
0.820001025433420 00	0.629160914881360-03

ABSCISSAS

WEIGHTS

8 POINT RULES, K=4 (CONTINUED)

I = 4

-0.833597023994930 00	0.108792182060900-02
-0.629436168619910 00	0.154617220197710-01
-0.387600431525230 00	0.748295265140300-01
-0.130699783459210 00	0.158620829645590 00
0.130699783459210 00	0.158620829645590 00
0.387600431525230 00	0.748295265140300-01
0.629436168619910 00	0.154617220197720-01
0.833597023994930 00	0.108792182060900-02

*** 9 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=4 ***

I = 1

-0.975764163139590 00	0.597242610825350-01
-0.874460105092310 00	0.114712225709490 00
-0.700827462777220 00	0.126428852545850 00
-0.468884753693080 00	0.101085771379270 00
-0.197406071600740 00	0.609691528219280-01
0.916389551265400-01	0.271752626124050-01
0.374885780355450 00	0.832360773473360-02
0.629533634360090 00	0.148279961406940-02
0.835569900746570 00	0.980664997222310-04

I = 2

-0.945597961398830 00	0.135856645756140-01
-0.821806586180320 00	0.624377473272970-01
-0.638671758996880 00	0.120700472474920 00
-0.410035175432570 00	0.138565979709210 00
-0.151508307821940 00	0.100743575139710 00
0.124073442517420 00	0.467919651420910-01
0.396644595179200 00	0.144267181217890-01
0.642260796412040 00	0.257719187697330-02
0.841177377125570 00	0.170685632396210-03

I = 3

-0.908032436980840 00	0.310153699398590-02
-0.760565123047050 00	0.263316946661750-01
-0.563995725340370 00	0.832401192842900-01
-0.333887291189500 00	0.140911175899880 00
-0.834149428583230-01	0.136750066601580 00
0.174947609603140 00	0.787802163090750-01
0.428955839474140 00	0.258987044630710-01
0.660857719606660 00	0.467544068570100-02
0.849310742838430 00	0.311045096241730-03

ABSCISSAS

WEIGHTS

9 POINT RULES, K=4 (CONTINUED)

I = 4

-0.859826644848130 00	0.551516909228530-03
-0.684989167844700 00	0.822192883321270-02
-0.471511500789350 00	0.442832840911980-01
-0.239968535390750 00	0.116742179922140 00
0.0	0.160402180488450 00
0.239968535390750 00	0.116742179922140 00
0.471511500789340 00	0.442832840911980-01
0.684989167844700 00	0.822192883321290-02
0.859826644848130 00	0.551516909228550-03

*** 10 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=4 ***

I = 1

-0.979681225122820 00	0.504020495490380-01
-0.894462953806160 00	0.998980699509710-01
-0.747248839774120 00	0.116822637070740 00
-0.548033148351370 00	0.102456339027320 00
-0.310382665985030 00	0.707750729108760-01
-0.504825185151700-01	0.384272106097300-01
0.213974480239940 00	0.158259059060620-01
0.465008670311430 00	0.456520403111170-02
0.685633266031240 00	0.777768688589290-03
0.861327108885090 00	0.498422555648720-04

I = 2

-0.953905288883320 00	0.989576140407840-02
-0.848464691454230 00	0.475027215639310-01
-0.690970475973820 00	0.990091799745630-01
-0.491596543403310 00	0.128080492949210 00
-0.262481762314030 00	0.112005243280350 00
-0.150091276078200-01	0.663732344470880-01
0.239511045175520 00	0.276662668196290-01
0.482080538872440 00	0.801180049904570-02
0.695563522820020 00	0.136756913346590-02
0.865681336983740 00	0.877299286342690-04

I = 3

-0.921646839867700 00	0.194999440008000-02
-0.794980198484490 00	0.173452949590790-01
-0.623916569863440 00	0.593056795824210-01
-0.420039759635700 00	0.114817933439810 00
-0.195284463144610 00	0.136465493308820 00
0.414627683690100-01	0.103264920868230 00
0.278841957163470 00	0.494772951930500-01
0.507502293478760 00	0.146866528822660-01
0.710174450886970 00	0.252426405697290-02
0.872049120862860 00	0.162471310265880-03

ABSCISSAS

WEIGHTS

10 POINT RULES, K=4 (CONTINUED)

I = 4

-0.880346632350700 00	0.294306717503760-03
-0.729268133095680 00	0.454505428898330-02
-0.540980088946180 00	0.260655966466390-01
-0.331571157173600 00	0.803537494496730-01
-0.111525268684020 00	0.138741292897200 00
0.111525268684020 00	0.138741292897200 00
0.331571157173600 00	0.803537494496730-01
0.540980088946180 00	0.260655966466390-01
0.729268133095680 00	0.454505428898340-02
0.880346632350700 00	0.294306717503760-03

*** 11 POINT RULES, WEIGHT FUNCTION V(I,K;T), K=4 ***

I = 1

-0.982717357115480 00	0.430896788206550-01
-0.910042301443030 00	0.875023134864470-01
-0.783730428425070 00	0.107077720355250 00
-0.611091117756160 00	0.100706733937000 00
-0.402151359807740 00	0.769063086413030-01
-0.169048827176180 00	0.480809076167690-01
0.746774616496410-01	0.241973940017320-01
0.314878631564860 00	0.939590215981530-02
0.537630658281520 00	0.258936259165190-02
0.730089542069360 00	0.426409607035620-03
0.881505098158260 00	0.266687823434690-04

I = 2

-0.960442843393380 00	0.737094103986630-02
-0.869585124380170 00	0.365988158184920-01
-0.732850932960480 00	0.808038528767660-01
-0.557852589817910 00	0.114275480650980 00
-0.353968426698370 00	0.114338267743120 00
-0.130983536160180 00	0.820402771214920-01
0.103017502182360 00	0.425579486662920-01
0.335355744400000 00	0.166176453333500-01
0.551264949202340 00	0.459226898660090-02
0.737983980419430 00	0.757112418398930-03
0.884953595777570 00	0.473893446397200-04

I = 3

-0.932451023800400 00	0.126574718578510-02
-0.822557671307900 00	0.116710125404900-01
-0.672643027456660 00	0.423250997146940-01
-0.491407381948590 00	0.901880228248790-01
-0.288894278355450 00	0.125106179600690 00
-0.726696178865590-01	0.115678665546130 00
0.148569578920480 00	0.730706122691290-01
0.366542277766730 00	0.306271515360030-01
0.571623525386880 00	0.856041316571090-02
0.749663901378300 00	0.141810886451830-02
0.890030051810830 00	0.889867519708340-04

ABSCISSAS

WEIGHTS

11 POINT RULES, K=4 (CONTINUED)

I = 4

-0.896696927670650 00	0.164150462262450-03
-0.765044668396780 00	0.260371565286640-02
-0.598594601259630 00	0.155650757160940-01
-0.408901597151900 00	0.532634155697210-01
-0.207428771117130 00	0.109107375455170 00
0.0	0.138592534287770 00
0.207428771117130 00	0.109107375455170 00
0.408901597151900 00	0.532634155697210-01
0.598594601259630 00	0.155650757160940-01
0.765044668396780 00	0.260371565286650-02
0.896696927670650 00	0.164150462262460-03

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*** 12 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=4 ***

I = 1

-0.985118846919450 00	0.372518583313930-01
-0.922411906059110 00	0.771086253196710-01
-0.812899713981700 00	0.977756473622580-01
-0.662047136413470 00	0.970308748840800-01
-0.477414075392580 00	0.800391686461920-01
-0.268255925156340 00	0.555894971464040-01
-0.450568946637960-01	0.323176198985600-01
0.180997530360530 00	0.153607827920270-01
0.398585176785050 00	0.570100447824150-02
0.596825480512420 00	0.151687869505050-02
0.765864453041190 00	0.243125820104820-03
0.897598524553890 00	0.149166260195660-04

I = 2

-0.965680026932260 00	0.559878939094110-02
-0.886594061731540 00	0.285540434106500-01
-0.766863467140730 00	0.659512597980890-01
-0.612281985938660 00	0.998850957965660-01
-0.430133755246590 00	0.110562786326790 00
-0.228454331378770 00	0.918099847819030-01
-0.145271313976660-01	0.569524557660480-01
0.204096484352460 00	0.273317984085170-01
0.415250093657120 00	0.101786566549060-01
0.607879209066560 00	0.271309771295870-02
0.772240677561590 00	0.435307739762260-03
0.900375435515060 00	0.267242128712730-04

ABSCISSAS

WEIGHTS

12 POINT RULES, K=4 (CONTINUED)

I = 3

-0.941167509334060 00	0.844949572954000-03
-0.844974849474120 00	0.801494432252630-02
-0.712705400682710 00	0.304194214340680-01
-0.550992247765510 00	0.695962520188490-01
-0.367852354009830 00	0.108375577092380 00
-0.170498238438340 00	0.117244096144970 00
0.349487252055200-01	0.906335520996940-01
0.240641488754660 00	0.497692930645760-01
0.440585925570220 00	0.190983853998310-01
0.624447715448190 00	0.512717400985780-02
0.781726598544740 00	0.825571142904970-03
0.904488839540830 00	0.507836973893770-04

I = 4

-0.909921442959940 00	0.951923680890940-04
-0.794284574058050 00	0.154180098676300-02
-0.646487291853160 00	0.950763575505410-02
-0.474735023992570 00	0.346923112543500-01
-0.290023009122300 00	0.810895074296140-01
-0.973649625277000-01	0.123073552206130 00
0.973649625277000-01	0.123073552206130 00
0.290023009122300 00	0.810895074296140-01
0.474795023992570 00	0.346923112543510-01
0.646487291853160 00	0.950763575505410-02
0.794284574058050 00	0.154180098676290-02
0.909921442959940 00	0.951923680890880-04

*** 13 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=4 ***

I = 1

-0.987051378101880 00	0.325192618378240-01
-0.932396210325800 00	0.683542401960150-01
-0.836575970953490 00	0.891735566808650-01
-0.703755648566740 00	0.924276932334170-01
-0.539736649578830 00	0.809146991689840-01
-0.351665606031640 00	0.609369166444390-01
-0.147819217705480 00	0.394776589623810-01
0.629557376679800-01	0.217176497607030-01
0.271433010199970 00	0.987447670831520-02
0.468512373658970 00	0.353669153411160-02
0.645606459807600 00	0.914947874652380-03
0.795050188080800 00	0.143535660559000-03
0.910635325641640 00	0.867173773336930-05

ABSCISSAS

WEIGHTS

13 POINT RULES, K=4 (CONTINUED)

I = 2

-0.969940893772570 00	0.432648600459140-02
-0.900490835164520 00	0.225468640623420-01
-0.794840739435030 00	0.539875635692190-01
-0.657470952424660 00	0.862961596300960-01
-0.494093768828440 00	0.103254418213940 00
-0.311219495815070 00	0.958265177683020-01
-0.115337905459610 00	0.689356501704030-01
0.880882229341560-01	0.388087409880820-01
0.290524659529250 00	0.177295874204830-01
0.482251982818400 00	0.636476010882340-02
0.654689141447640 00	0.164874443009000-02
0.800273161109410 00	0.258860761630980-03
0.912904417536160 00	0.156468719976240-04

I = 3

-0.948300267245370 00	0.578206680534310-03
-0.863428996256670 00	0.561095232878950-02
-0.745982626137410 00	0.220756708000660-01
-0.601092598116040 00	0.533406028980240-01
-0.435097767904260 00	0.904896559442030-01
-0.254581616070230 00	0.111065296633810 00
-0.643794116444680-01	0.100189389716370 00
0.129196838722170 00	0.675643930442660-01
0.320251133267180 00	0.332980753201830-01
0.503138231585730 00	0.121099445453260-01
0.668346400155450 00	0.315246267957100-02
0.808077950975460 00	0.496302715789810-03
0.916282552121630 00	0.300467030735670-04

I = 4

-0.920768781884450 00	0.571167676249860-04
-0.818465061350620 00	0.940634780162750-03
-0.686594558050200 00	0.594281109979270-02
-0.531299389842370 00	0.225642390196250-01
-0.361406071310070 00	0.581828399013600-01
-0.182819277110170 00	0.101258551039030 00
0.0	0.122107614784810 00
0.182819277110170 00	0.101258551039030 00
0.361406071310070 00	0.581828399013610-01
0.531299389842370 00	0.225642390196250-01
0.686594558050190 00	0.594281109979300-02
0.818465061350620 00	0.940634780162800-03
0.920768781884450 00	0.571167676249910-04

ABSCISSAS

WEIGHTS

*** 14 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=4 ***

I = 1

-0.988629791436720	00	0.286307692594730	-01
-0.940571079754230	00	0.609389009316380	-01
-0.856049258105540	00	0.813558829865240	-01
-0.738292571675330	00	0.873005833211110	-01
-0.591823506718190	00	0.801820434039240	-01
-0.422269577422890	00	0.643689528421020	-01
-0.236145479149110	00	0.453489455970070	-01
-0.406022144006580	-01	0.278556183983370	-01
0.156848282989720	00	0.146753970903780	-01
0.348622941930020	00	0.644125002019370	-02
0.527361688289760	00	0.224203760220950	-02
0.686218242105610	00	0.566991724286800	-03
0.819153081271890	00	0.874117597851210	-04
0.921340532398220	00	0.521506303092770	-05

I = 2

-0.973453926935850	00	0.339468651839310	-02
-0.911987960700520	00	0.180063402508780	-01
-0.818114021921080	00	0.443964821370130	-01
-0.695347311208310	00	0.741045375388520	-01
-0.548215325015290	00	0.942823127508990	-01
-0.381986958368290	00	0.953738212880850	-01
-0.202203847047750	00	0.772200602555880	-01
-0.138129976977940	-01	0.498433677409170	-01
0.177865774895020	00	0.264670093450240	-01
0.364582324360800	00	0.116519165511670	-01
0.538816895997830	00	0.406245622612650	-02
0.693768750500050	00	0.102839429987320	-02
0.823483778274520	00	0.158646310405740	-03
0.923218203199780	00	0.946878677851810	-05

I = 3

-0.954211319881660	00	0.404468843809570	-03
-0.878796426355530	00	0.399852130458690	-02
-0.773894872475460	00	0.161944857984840	-01
-0.643532243584470	00	0.408492473035810	-01
-0.492749273752110	00	0.739321527218670	-01
-0.327168575680420	00	0.100269177339400	00
-0.151314147548920	00	0.102444780150620	00
0.302024900601100	-01	0.807362399263980	-01
0.211819483880410	00	0.487856744174810	-01
0.389269498701440	00	0.222562201665460	-01
0.556251009267260	00	0.781735814394180	-02
0.705159973190490	00	0.198625243134660	-02
0.829982868971090	00	0.307070284267820	-03
0.926026848314290	00	0.183511676800120	-04

ABSCISSAS

WEIGHTS

14 POINT RULES, K=4 (CONTINUED)

I = 4

-0.929774554429820 00	0.353198248564650-04
-0.838671954852540 00	0.589560224158070-03
-0.720442607503930 00	0.379736442407870-02
-0.579803139733730 00	0.148249375861870-01
-0.423417970794260 00	0.408737492635870-01
-0.257867853344350 00	0.793994177659190-01
-0.864508853097470-01	0.110479450911210 00
0.864508853097470-01	0.110479450911210 00
0.257867853344350 00	0.793994177659190-01
0.423417970794260 00	0.408737492635870-01
0.579803139733730 00	0.148249375861870-01
0.720442607503920 00	0.379736442407900-02
0.838671954852540 00	0.589560224158080-03
0.929774554429820 00	0.353198248564650-04

*** 15 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=4 ***

I = 1

-0.989935751362140 00	0.253977267311800-01
-0.947348581773790 00	0.546194779133990-01
-0.872253917784200 00	0.743185041256800-01
-0.767191512018450 00	0.820479215239490-01
-0.635736747556890 00	0.783564268919430-01
-0.482365276593380 00	0.662232473062100-01
-0.312299258230290 00	0.498660250138130-01
-0.131329168964840 00	0.333742651889970-01
0.543836637293720-01	0.196571251078110-01
0.238517332268030 00	0.100033110958350-01
0.414805599587900 00	0.426689487687450-02
0.577253677325660 00	0.145095418002270-02
0.720349379021100 00	0.360155770173690-03
0.839277195663680 00	0.547325981985160-04
0.930237003051670 00	0.323167591450670-05

I = 2

-0.976384710897600 00	0.269998985565550-02
-0.921606736917400 00	0.145328596331950-01
-0.837673010938280 00	0.367072738075420-01
-0.727378073076410 00	0.634728119431090-01
-0.594348971902530 00	0.848458301371610-01
-0.442868416936810 00	0.918778094746560-01
-0.277588792330590 00	0.816532855549680-01
-0.103020477618800 00	0.592984187771240-01
0.769465219052480-01	0.355638678357370-01
0.256289334042300 00	0.181726682682400-01
0.428280470374500 00	0.776771948462070-02
0.586901494962490 00	0.264468830609700-02
0.726692616903390 00	0.656983877550720-03
0.842907559664680 00	0.998929275652520-04
0.931808399167340 00	0.590011677820740-05

ABSCISSAS

WEIGHTS

15 POINT RULES, K=4 (CONTINUED)

I = 3

-0.95916398023943D 00	0.28855725906533D-03
-0.89172322752491D 00	0.28967470242629D-02
-0.79751267867019D 00	0.12013955975030D-01
-0.67973069273879D 00	0.31370701459583D-01
-0.54242191792662D 00	0.59714229166119D-01
-0.39022956777050D 00	0.87464885087152D-01
-0.22753529034234D 00	0.99135109679627D-01
-0.57785633230588D-01	0.88330162046415D-01
0.11434178415081D 00	0.62553101243298D-01
0.28451057241603D 00	0.34592068090420D-01
0.44907741054414D 00	0.15015153512107D-01
0.60160726307505D 00	0.51386512879259D-02
0.73629114090979D 00	0.12801651889829D-02
0.84837587589283D 00	0.19498402469660D-03
0.93416852033068D 00	0.11528955316360D-04

I = 4

-0.93733082101334D 00	0.22437123490558D-04
-0.85571588629596D 00	0.37869524580226D-03
-0.74921410698583D 00	0.24778348165222D-02
-0.62151633423286D 00	0.98860289546386D-02
-0.47764498025382D 00	0.28435157686142D-01
-0.32392578056320D 00	0.60320099151074D-01
-0.16350495681058D 00	0.93890794838073D-01
0.0	0.10917790436851D 00
0.16350495681058D 00	0.93890794838074D-01
0.32392578056320D 00	0.60320099151074D-01
0.47764498025381D 00	0.28435157686143D-01
0.62151633423285D 00	0.98860289546391D-02
0.74921410698582D 00	0.24778348165224D-02
0.85571588629596D 00	0.37869524580230D-03
0.93733082101334D 00	0.22437123490563D-04

*** 16 POINT RULES, WEIGHT FUNCTION N(I,K;T), K=4 ***

I = 1

-0.99102863372480D 00	0.22681189044200D-01
-0.95302985923236D 00	0.49200763407361D-01
-0.88587952992211D 00	0.68014741099320D-01
-0.79160242654178D 00	0.76881311098915D-01
-0.67306104904876D 00	0.75825274380237D-01
-0.53385653544128D 00	0.66839307123605D-01
-0.37821806576490D 00	0.53115296195032D-01
-0.21087408947224D 00	0.38065556291102D-01
-0.36908496146690D-01	0.24460583616674D-01
0.13839406404629D 00	0.13927166350090D-01
0.30970914164911D 00	0.68897019420471D-02
0.47183511928892D 00	0.28704798981407D-02
0.61985337693011D 00	0.95743612166766D-03
0.74928396502980D 00	0.23399936886880D-03
0.85624517500379D 00	0.35137308296773D-04
0.93770927191831D 00	0.20567544422669D-05

ABSCISSAS

WEIGHTS

16 POINT RULES, K=4 (CONTINUED)

I = 2

-0.578655180448500 00	0.217377232421020-02
-0.929733900084510 00	0.118450978290380-01
-0.854261041226010 00	0.305267845193290-01
-0.754684899414440 00	0.543513932250100-01
-0.633940996854020 00	0.756486579480120-01
-0.495532493799510 00	0.865389945715730-01
-0.343345622773700 00	0.828276420852970-01
-0.181342809892100 00	0.663068480471880-01
-0.130466207908500-01	0.442955797334270-01
0.157640884906690 00	0.253867516526280-01
0.324873079512720 00	0.125924729660750-01
0.483311668692070 00	0.525446603426080-02
0.628051899064520 00	0.175429109873700-02
0.754662700272420 00	0.429024728466930-03
0.859317899289880 00	0.644496455270680-04
0.939037430922450 00	0.377359122233080-05

I = 3

-0.963354773614720 00	0.209531931145500-03
-0.902697424205540 00	0.213060621053510-02
-0.817660795894350 00	0.901222195505180-02
-0.710815671125710 00	0.242081547963930-01
-0.585438602914870 00	0.479740462775180-01
-0.445347778403030 00	0.745614982761850-01
-0.294535847599730 00	0.920994962772280-01
-0.136022929915260 00	0.907809912386650-01
0.265879232309740-01	0.727788103338070-01
0.189238452159530 00	0.471861399816850-01
0.348694796726010 00	0.243873915569550-01
0.501008828432500 00	0.102569409493090-01
0.640569328160980 00	0.343744619036590-02
0.762824436590080 00	0.842545814101030-03
0.863961854951970 00	0.126750203922430-03
0.941039584788190 00	0.742800713314360-05

I = 4

-0.943732355345760 00	0.146008665402100-04
-0.870217905787690 00	0.248718355845040-03
-0.773849063968600 00	0.164867305707330-02
-0.657555699787010 00	0.669470793353050-02
-0.525254774656520 00	0.197887756883100-01
-0.382245561689120 00	0.448709974274600-01
-0.232207005990300 00	0.765664191938970-01
-0.777687935194040-01	0.100167107477340 00
0.777687935194040-01	0.100167107477340 00
0.232207005990300 00	0.765664191938970-01
0.382245561689120 00	0.448709974274610-01
0.525254774656520 00	0.197887756883100-01
0.657555699787000 00	0.669470793353070-02
0.773849063968600 00	0.164867305707350-02
0.870217905787680 00	0.248718355845070-03
0.943732355345750 00	0.146008665402130-04

ABSCISSAS

WEIGHTS

*** 17 POINT RULES, WEIGHT FUNCTION $W(I, K; T)$, $K=4$ ***

I = 1

-0.991952456285190 00	0.203770490080380-01
-0.957839116245210 00	0.445262875154530-01
-0.897443682684980 00	0.623801444131540-01
-0.812399675723780 00	0.719232246356350-01
-0.705025066066810 00	0.728701561471930-01
-0.578248202048880 00	0.665184779164960-01
-0.435526888959340 00	0.552552146271970-01
-0.280753847335610 00	0.418631752406960-01
-0.118150428227930 00	0.288424349218650-01
0.478486404931190-01	0.179321080953480-01
0.212716396634780 00	0.992988404314340-02
0.371957470795350 00	0.479846359648720-02
0.521231465352100 00	0.196037475412650-02
0.656473238802880 00	0.643380877089760-03
0.774009357109350 00	0.155207718388260-03
0.870679290550200 00	0.230718517062650-04
0.944045125461520 00	0.134064798320840-05

I = 2

-0.980957049874110 00	0.176941163822610-02
-0.936662213356200 00	0.974254602366720-02
-0.868446880388630 00	0.255379851271110-01
-0.778139620469960 00	0.465960720327500-01
-0.668139268062030 00	0.670649735122220-01
-0.541331820122040 00	0.802450724865060-01
-0.400966094591390 00	0.815432086670000-01
-0.250460524944910 00	0.706465118742160-01
-0.930735758414040-01	0.519419140108880-01
0.683112227797460-01	0.327659059450590-01
0.229283889961390 00	0.182092639662370-01
0.384999594213620 00	0.881604103373790-02
0.531083964726840 00	0.360591724231090-02
0.663497398662280 00	0.118434157982530-02
0.778609051996990 00	0.285856255407900-03
0.873302865887010 00	0.425079925584780-04
0.945177795143920 00	0.247061227577940-05

ABSCISSAS

WEIGHTS

17 POINT RULES, K=4 (CONTINUED)

I = 3

-0.966932297512280 00	0.154593399802730-03
-0.912091473462660 00	0.158911477732750-02
-0.834978131045240 00	0.683384159192910-02
-0.737680581167750 00	0.187930862378180-01
-0.622878041448580 00	0.384844627089970-01
-0.493723719754950 00	0.626737293838900-01
-0.353665389897820 00	0.829595558473960-01
-0.205670301194030 00	0.891202107827560-01
-0.524151453751940-01	0.789639726232120-01
0.102579479222600 00	0.580863260212470-01
0.256046464489840 00	0.349906918013180-01
0.405394154320460 00	0.172608008804920-01
0.546277934877060 00	0.709734710431420-02
0.674240919374700 00	0.233789655367240-02
0.785606957463720 00	0.565311228350190-03
0.877280216481260 00	0.841636522819560-04
0.946890960851390 00	0.489540519743390-05

I = 4

-0.949202920667520 00	0.970951789510850-05
-0.882655849208280 00	0.166684875040560-03
-0.795088010632070 00	0.111697018245720-02
-0.688854665904070 00	0.460188346642460-02
-0.567097570848940 00	0.138758077727820-01
-0.433950901444360 00	0.329461764747910-01
-0.293540933533510 00	0.606961334887920-01
-0.147924813816180 00	0.872098649821340-01
0.0	0.987535384793630-01
0.147924813816180 00	0.872098649821340-01
0.293540933533510 00	0.606961334887920-01
0.433950901444360 00	0.329461764747920-01
0.567097570848940 00	0.138758077727830-01
0.688854665904060 00	0.460188346642470-02
0.795088010632070 00	0.111697018245730-02
0.882655849208270 00	0.166684875040590-03
0.949202920667520 00	0.970951789511190-05